

Perspectives on Finding the Neutrino Nature

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(February 1, 2008)

The possibility of determining the neutrino nature is considered in view of the most recent experimental observations. The analysis combines schemes with three and four neutrinos. The data on oscillations is put together with that from the search of neutrinoless double beta decay and results on tritium beta decay. All solar neutrino oscillation solutions are taken into account. The sensitivity of the problem on future experimental bounds from GENIUS is studied. Dirac neutrinos are shown to be unavoidable already at present in some schemes and the constraints will quickly become more stringent with future data. The consequences of including bounds from Cosmology on the neutrino content of Hot Dark Matter are commented.

At present, particle physics seems to have a long awaited opportunity to discover the path towards an extension of the Standard Model. This opportunity is given by the recent confirmation of neutrino oscillations [1]. With the currently accumulated experimental data, we have many hints on the size of neutrino masses and mixing angles. This information imposes serious constraints on the structure of the model. Unfortunately there remains a major unknown, which is the neutrino nature. As a neutral particle it can be described either by means of the Dirac equation or by means of the Majorana equation [2]. This problem is of the utmost importance for model builders.

It has been proved many times, that with the current precision, the only experiment able to show which of the two descriptions is valid, is the search for the neutrinoless double beta decay $(\beta\beta)_{0\nu}$. Should this process be observed, we would be left with no other choice but to acknowledge neutrinos to be Majorana particles. As this has not been the case, we have at hand only a lower bound on the lifetime of the $(\beta\beta)_{0\nu}$ decaying nuclei. This alone is not enough. However when supplied with the results of neutrino oscillation experiments and the tritium beta decay mass bounds, interesting, but this time neutrino mass scheme dependent conclusions can be drawn. The situation will undoubtedly improve, when the future GENIUS [3] experiment will furnish its first results.

In this paper, we combine the neutrino oscillation data, the tritium beta decay bounds and the present and future GENIUS bounds on the $(\beta\beta)_{0\nu}$ lifetime (if their results are not positive, since in the opposite case the situation is clear, as stated above) in order to derive conditions which would define the neutrino as a Dirac particle. To this end we take into account the possibility of having three or four light neutrinos. Therefore our conclusions will be valid in either case, whether the LSND [4] results would eventually be ruled out or confirmed.

The novelty in respect to previous analyses [5] is the consideration of the case of three and four neutrinos together, supplemented with the most recent bounds on the mixing matrix elements and the variation of the minimal

neutrino mass $(m_\nu)_{min}$ in its full range. Furthermore we do not assume that neutrinos are Majorana particles like in so many other works [6].

The oscillation data is described in terms of differences of neutrino mass squares $\delta m_{ij}^2 \equiv m_i^2 - m_j^2$. Define a scheme of neutrino masses as an ordering of these δm^2 's. For three neutrinos there are two such schemes, whereas for four neutrinos there are six. In the latter case, four schemes can be rejected by means of a combined analysis [7]. All four possibilities are shown in Fig. 2 and Fig 3. It should be clear that with the strong ordering of δm^2 's ($\delta m_{sol}^2 \ll \delta m_{atm}^2 \ll \delta m_{LSND}^2$), it is only the largest that actually matters. Therefore for three neutrinos we need only to know $\delta m_{atm}^2 \approx 3 \times 10^{-3} \text{ eV}^2$ [8], whereas for four $\delta m_{LSND}^2 \approx 1 \text{ eV}^2$ [4,9]. The second important information is that the spectrum of the masses of light neutrinos is bounded from above by $m_\nu < 2.5 \text{ eV}$ [10]. With the additional information on the value of one mass, we recover the complete mass spectrum of neutrinos.

The basic quantity to consider is the effective neutrino mass, measured in $(\beta\beta)_{0\nu}$ decay

$$|\langle m_\nu \rangle| \equiv \left| \sum_{i=light} U_{ei}^2 m_i \right|, \quad (1)$$

with U the mixing matrix which we assume unitary. Current and future bounds on $|\langle m_\nu \rangle|$ are given in Table I. At present, nothing can be said about the phases of U , therefore we allow them to have the least favorable values. Our analysis is based on the following statement: if the minimum of $|\langle m_\nu \rangle|$ with respect to U_{ei} within their allowed range, $(|\langle m_\nu \rangle|)_{min}$, exceeds the experimental bound, then neutrinos are Dirac particles [12].

Suppose that the solar neutrino oscillations take place mostly between the first and the second neutrino states, then [13]

$$(|\langle m_\nu \rangle|)_{min} = \min |l^e m_a - s^e m_b|, \quad (2)$$

with

$$l^e \in [l_-^e, l_+^e], \quad s^e \in [s_-^e, s_+^e], \quad (3)$$

TABLE I. present and future bounds on the effective neutrino mass (in eV) from $(\beta\beta)_{0\nu}$ decay search.

$ \langle m_\nu \rangle <$	experiment
.2	Heidleberg-Moscow [11]
2×10^{-2}	GENIUS, first stage, 1yr, 1t [3]
6×10^{-3}	GENIUS, second stage, 10yr, 1t [3]

and

$$l_+^e = (1 - s_+^e), \quad l_-^e = (1 - s_+^e)\sqrt{1 - \sin^2 2\theta_{\text{sun}}}, \quad (4)$$

where in the case of three neutrinos

$$s_+^e = s_-^e = |U_{e3}|^2, \quad (5)$$

and in the case of four

$$s_+^e = |U_{e3}|^2 + |U_{e4}|^2, \quad s_-^e = ||U_{e3}|^2 - |U_{e4}|^2|. \quad (6)$$

The mass parameters in Eq. 2 can be chosen in two different ways, depending on the ordering of masses of the neutrino states.

Schemes A_3 and A_4 . The behavior of the minimum of the effective mass is the following

$$(|\langle m_\nu \rangle|)_{\min} = \begin{cases} s_-^e \sqrt{(m_\nu)_{\min}^2 + \delta m^2} - l_+^e (m_\nu)_{\min}, \\ 0, \\ l_-^e (m_\nu)_{\min} - s_+^e \sqrt{(m_\nu)_{\min}^2 + \delta m^2}, \end{cases} \quad (7)$$

with $\delta m^2 \equiv \delta m_{\text{atm}}^2$ for three neutrinos and $\delta m^2 \equiv \delta m_{\text{LSND}}^2$ for four. The third range occurs only if $l_-^e > s_+^e$. This can be rewritten as

$$\sin^2 2\theta_{\text{sun}} < \frac{1 - 2s_+^e}{(1 - s_+^e)^2}, \quad (8)$$

Obviously these schemes always allow for a Majorana mass. If however the condition Eq. 8 is satisfied, then it is possible to derive a lower bound on the mass spectrum which excludes this possibility. This bound will be larger than the value of $(m_\nu)_{\min}$ which marks the beginning of the third range in Eq. 7

$$(m_\nu)_{\min} = \left(\frac{\delta m^2 (s_+^e)^2}{(l_-^e)^2 - (s_+^e)^2} \right)^{1/2}. \quad (9)$$

This number will be small in the case of three neutrinos if $\sin^2 2\theta_{\text{sun}}$ is not too close to one, due to the small δm_{atm}^2 , but can quickly grow large in the case of four.

Schemes B_3 and B_4 . At the condition Eq. 8 we have

$$(|\langle m_\nu \rangle|)_{\min} = l_-^e \sqrt{(m_\nu)_{\min}^2 + \delta m^2} - s_+^e (m_\nu)_{\min}. \quad (10)$$

Obviously, if the masses of the neutrinos are distributed according to these schemes the neutrino nature can be determined irrespective of the lowest mass, depending solely on the experimental bound on $|\langle m_\nu \rangle|$

Dependence on the mixing matrix. The condition Eq. 8 plays a crucial role in the problem. The availability of lower bounds on the neutrino mass is tightly connected to it. We therefore have to investigate the size of both sides of the inequality.

The quantity s_+^e can be fitted to various experimental data. It turns out that the solar neutrino flux observation is the least constraining since one has a bound of $s_+^e < .67 - .73$ [14]. Remark however that the difference between χ^2 for $s_+^e = 0$ and the above result is almost irrelevant. Therefore if other experiments constrain the considered quantity to be close to zero, then solar neutrino oscillation should be viewed as a two state phenomenon and 3 neutrino fits to solar data alone seem to be of little value. This will be important subsequently. The currently strongest constraint comes from a combined analysis of atmospheric and reactor data. Namely one has $s_+^e < .01 - .02$ [15]. However a recent fit to the new (830-920 days) data of Super-Kamionkande [16] gives a best value of $s_+^e \simeq .03$. There has also been some speculation about strengthening this bound by adding some theoretical assumptions on the shape of the mass matrix. It seems that $s_+^e \simeq 0$ is the most preferred value [17]. In Fig. 1, the dependence of the upper bound on $\sin^2 2\theta_{\text{sun}}$ is shown as function of s_+^e . In the range $s_+^e \in [0, .04]$ this bound is between .998 and 1.

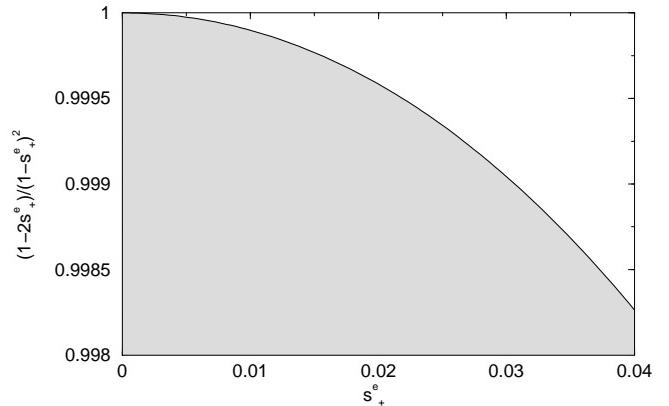


FIG. 1. Allowed region of $\sin^2 2\theta_{\text{sun}}$ by condition Eq. 8.

Let us now turn to the experimental bounds on $\sin^2 2\theta_{\text{sun}}$. Various fits have been performed for two [18], three [14,19] and even four [20] neutrino oscillation scenarios. The most recent which take into account the full data (rates, energy spectrum, day-night asymmetry in the case of the MSW solution, and the seasonal variation in the case of the VO solution) give a best fit value of $\sin^2 2\theta_{\text{sun}} < 1$. The 95% c.l. bounds for the MSW solution are given in Table II. The most important conclusions are that in neither case do $\sin^2 2\theta_{\text{sun}}$ exceed the bound given by condition Eq. 8.

For the Vacuum Oscillation solution best fit value we take $\sin^2 2\theta_{\text{sun}} = .93$ [21]. The above values have been used to draw Fig. 2 and Fig. 3 for the case $s_+^e = .01$.

TABLE II. Allowed ranges and best fit values of $\sin^2 2\theta_{sun}$ for the different types of oscillations [14].

oscillation type	95% <i>c.l.</i> range	best fit
SMA	.001 – .01	.0065
LMA	.58 – .98	.77
LOW	.68 – .98	.90

Discussion. The most stringent of the oscillation solutions when question is of the neutrino nature, is the SMA. Already with the present bound one must adopt the Dirac description in the B_4 scheme. With negative results of GENIUS the same will hold true of the B_3 scheme. Currently a bound of $m_\nu > .22\text{eV}$ can be taken as a delimiter of the two neutrino natures for the A_3 , A_4 and B_3 schemes.

At the 95%*c.l.* the remaining solutions LMA and LOW define the neutrino nature if $m_\nu > 1.5\text{eV}$, with the present bound and the A_3 , A_4 , B_3 schemes and $m_\nu > 1.1\text{eV}$ in B_4 . With the GENIUS I bound this can be shifted down to $m_\nu < .16\text{eV}$ (A_3), $m_\nu < .14\text{eV}$ (B_3) and $m_\nu < .22\text{eV}$ (A_4). Finally GENIUS II would give $m_\nu < .05\text{eV}$ for the A_3 and $m_\nu < .12\text{eV}$ for the A_4 scheme, but would completely exclude a Majorana neutrino in the B_3 scheme.

We do not have a 95%*c.l.* bound for the VO solution. However, with the assumption that here also $\sin^2 2\theta_{sun} < .98$ the same conclusions would hold as above.

If one takes the best fit values for the different solutions, all of the bounds can be improved.

An interesting consideration is to take into account the current best fit for the Hot Dark Matter neutrino constituent. One then has $\sum_{light} m_\nu \simeq 2\text{eV}$ [22]. The SMA solution rules out in this case Majorana neutrinos in the A_3 and B_3 schemes. With a negative result from GENIUS all solutions share this property in the A_3 , B_3 and B_4 schemes. Obviously, if we take the previous result $\sum_{light} m_\nu \simeq 4 – 5\text{eV}$ [23], then the constraint is even stronger as already now most of the schemes are ruled out and GENIUS would close the situation.

Final considerations. Several points must be stressed. First, the bound on $|\langle m_\nu \rangle|$ depends strongly on the precise determination of nuclear matrix elements. It is acknowledged that they suffer from an imprecision at the level of a factor of 3 [24]. Second, the confidence level that we assumed, 95% corresponds to 2σ . At 3σ which is 99%*c.l.*, the $\sin^2 2\theta_{sun}$ value of one is allowed. This unfortunately ruins the validity of condition Eq. 8 with the same effect on our predictions. Third, the figures have been drawn for $\delta m_{LSND}^2 = 1\text{eV}^2$. The experimentally allowed range is $\delta m_{LSND}^2 = .2 – 2\text{eV}^2$. Adoption of some other difference of mass squared would require a rescaling of the results for the A_4 and B_4 neutrino schemes.

Conclusions. The nature of neutrinos that seemed so evasive is closer and closer to the reach of experiments.

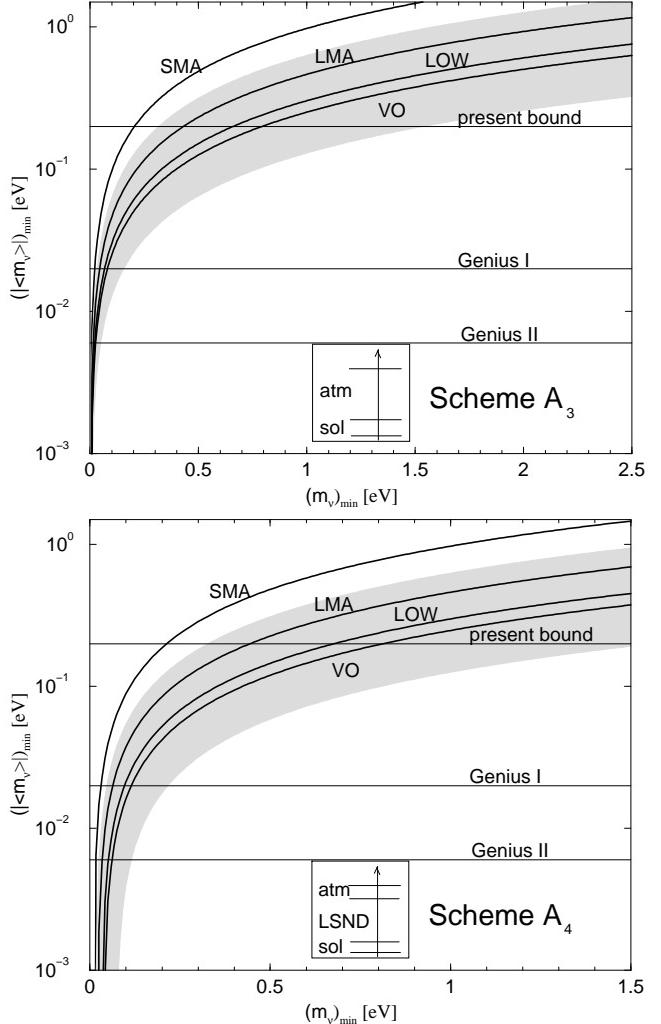


FIG. 2. Schemes A_3 and A_4 . The curves for the different solar neutrino oscillation solutions were obtained with the values of $\sin^2 2\theta_{sun}$ quoted in the text and $s_+^e = .01$. The shaded region is given by the largest range in Table II, that is the range for the LMA solution. For the SMA solution the 95%*c.l.* range (.001-.01) is described by one curve in the present scale.

Combining oscillation data, tritium beta decay and neutrinoless double beta decay is a powerful method of constraining the parameter space in which Majorana neutrinos are allowed. In this paper we have derived bounds on the masses in several schemes and shown which schemes are or will be excluded out of consideration if neutrinos are to be truly neutral particles. We still have to wait for MiniBoone or SNO, to decide on the existence of the sterile neutrino, and for SNO, Borexino, Kamland to determine the solar neutrino solution and finally for more statistics in all the experiments to constrain more precisely the $\sin^2 2\theta_{sun}$.

Acknowledgments. This work was supported by the Polish Committee for Scientific Research under Grant No. 2P03B05418. J.G. would like to thank the Alexander

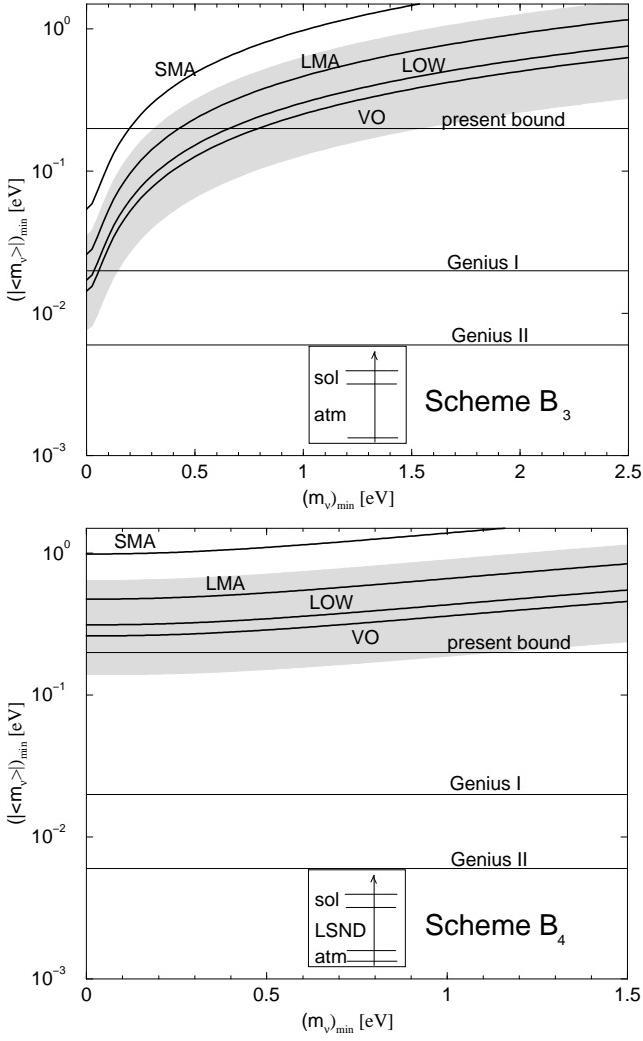


FIG. 3. Schemes B_3 and B_4 . The curves were obtained with the same assumptions as in Fig. 2.

von Humboldt-Stiftung for fellowship.

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